Social Value of Public Information under Asymmetric Precision of Private Signals: Implications for Capital Flows

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Motivation: Capital Flows to EMEs

> "A dollar-denominated government bond issued in 2012 by Zambia, a copper-rich country with an average GDP per person of $1,700 a year, offered just 5.4% interest; even so, it was 24 times oversubscribed as rich-world investors clamoured to buy. The following year a state-backed tuna-fishing venture in Mozambique, a country even poorer than Zambia, was able to raise $850m at an interest rate of 8.5%"
>
> –The Economist, *"Pulled back in"*, Nov 14th 2015

Following the aftermath of the Great Recession, many EMEs started receiving capital flows to their economies

- Good opportunity for EMEs to deal with capital scarcity
- Many policymakers viewed these flows as being "excessive" and introduced capital controls

But what kind of countries did receive these capital flows? Were these flows efficient relative to the intermediation capacity of domestic financial systems?

Literature on Global Financial Cycle

Helene Rey and co-authors (2013, 2014) introduce the literature on Global Financial Cycle

- Tendency for the prices of risky assets, credit growth and leverage around the world to move together
- A single "global factor" explains about a quarter of the variance of the prices of risky assets

We do not know what the global factor is BUT it is related to uncertainty and risk-aversion proxied by the VIX which in turn also related to US monetary policy (Bruno & Shin (2013))

What is VIX?

VIX is a measure of the implied volatility of S&P 500 index options

- Measures the market's expectation of stock market volatility over the next 30-day period
- Also usually considered as a measure of global risk aversion and risk appetite of investors (global financial factor)
- Reacts to different kinds of news (not only related to the US)

Effects of VIX on Capital Flows

Empirical literature on Capital Flows is voluminous

Generally, the literature agrees that VIX becomes one of the most robust statistically significant determinants of capital flows

- Negative relationship
- Robust result especially in the post-crisis period for EMEs
  - Ahmed and Zlate (2014): net flows for 12 EMEs
  - Forbes and Warnock (2012): gross flows in times of unusual flows for 58 countries (both EMEs and Advanced Economies)
  - IMF WEO, April 2016: VIX is the biggest contribution among global factors for 22 EMEs

But we do not know why and how this mechanism works

- What are the welfare implications?

My Approach

I want to study welfare implications for capital flows and cross-country spillovers using theories on information asymmetries

- Unobservable state of the economy – investors receive signals and update on them
- Countries generate lots of signals about the states of their economies
  - FX reserves
  - Financial development
  - Capital controls
  - Exchange rate regime
  - Growth prospects

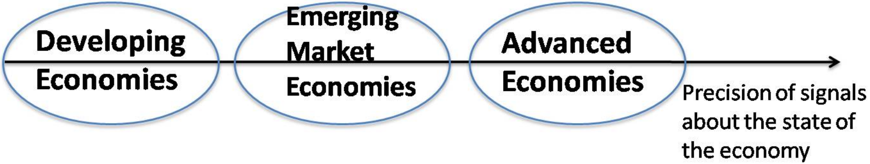
Relations to other approaches:

- Acharya and Bengui (2015): New Keynesian type of model with nominal rigidities and tradable and non-tradable sectors in continuous time
- Devereux et al. (2015): double-decker structure, spillover effects of interest rate shocks and financial shocks in advanced economies on EMEs

My Approach Cont'd

Investors make capital flow decisions using the information received from two sources

- Private signals: knowledge about fundamentals, experience from previous ventures
- Public signals: take into account a global factor (call it VIX)



Theoretical Literature on Overreaction to Public Signals

Morris and Shin (2002, AER)

- *Result 1:* Welfare is always increasing in the precision of private signals
- *Result 2:* Public signals can be bad for welfare because agents overreact to the public signal and if the coordination motive is relatively high enough ($r > 0.5$), then agents can coordinate at the expense of choosing actions that are further away from the true state. Private signals must not be very precise in order for increased precision of public information to be beneficial. Formally, if $\kappa_y > (2r-1)(1-r)\kappa_x$, then $\frac{\partial \mathbb{E}[W|\theta]}{\partial \kappa_x} > 0$

Research Question of the Theoretical Part

In Morris and Shin (2002, AER) there is no heterogeneity in the precision of private signals

- All agents are of the same type

How would the results about the social value of public information be different if we add this heterogeneity?

Analogy

- Type I agents (Advanced economies) have relatively less precise private signals about the state of the economy of type II agents (EMEs), while they both have private signals of the same precision about the state of the economy of type I agents

Overview of My Model

I follow Morris and Shin (2002) to build the model

- Game with continuous actions, strategic complementarity, and dispersed information
- Strategic behavior

How my approach is different

- Add one more fundamental and, hence, the additional choice variable in the actions' set of agents
- Add a second type of agents with a different precision of a private signal about one fundamental but with the same precision about another fundamental
- Public signal carries information about two fundamentals now

Setup

- There is a continuum of agents indexed by $i \in [0,1]$ of two types: $\lambda > 0.5$ measure of type I and $1-\lambda$ measure of type II
- Each type $k \in \{1,2\}$ of agents chooses two types of actions $a^1_{ki} \in \mathbb{R}$ and $a^2_{ki} \in \mathbb{R}$ to maximize expected payoffs

$$U_{ki}(a_{ki},\mathbf{a},\theta) = -(1-r)(a^1_{ki}-\theta^1)^2 - r(L^1_{ki}-\bar{L}^1) - (1-r)(a^2_{ki}-\theta^2)^2 - r(L^2_{ki}-\bar{L}^2)$$

where $k \in \{1,2\}$ denotes the type of the agent i and the superscript denotes fundamentals (1 stands for θ^1 and 2 stands for θ^2), $\mathbf{a}$ is the action profile over all agents, θ^1 and θ^2 are two fundamentals, $r \in [-1;1]$ governs the strategic interaction between agents and for $h \in \{1,2\}$

$$L^h_{ki} \equiv \int (a^h_{ki} - a^h_{.j})^2 dj$$

$$\bar{L}^h \equiv \int L^h_{.j} dj$$

Timing

- Nature draws θ^1 and θ^2 from improper uniform priors over the real line
- Agents receive two types of exogenous private signals

$$x_{.i}^1 = \theta^1 + \nu_{.i}^1 \quad \text{with} \quad \nu_{.i}^1 \sim N\left(0, \left(\kappa_{.x}^1\right)^{-1}\right)$$
$$x_{ki}^2 = \theta^2 + \nu_{ki}^2 \quad \text{with} \quad \nu_{ki}^2 \sim N\left(0, \left(\kappa_{kx}^2\right)^{-1}\right)$$

  Assume $\kappa_{.x}^1 > \kappa_{2x}^2 > \kappa_{1x}^2$.
- Agents receive an exogenous public signal which is a weighted average of two fundamentals:

$$y = \lambda\theta^1 + (1-\lambda)\theta^2 + \varepsilon \quad \text{with} \quad \varepsilon \sim N\left(0, \left(\kappa_y\right)^{-1}\right)$$

- Once agents observe their signals, agents choose their actions and payoffs are realized

Agent's Best Response Function

- For this game we find a Bayesian Nash equilibrium
- Each type of agents $k \in \{1,2\}$ solves

$$\max_{a^1_{ki}, a^2_{ki}} \mathbb{E}\left[-(1-r)(a^1_{ki}-\theta^1)^2 - r\left(\int (a^1_{ki}-a^1_{.j})^2 dj - \bar{L^1}\right) - (1-r)(a^2_{ki}-\theta^2)^2 - r\left(\int (a^2_{ki}-a^2_{.j})^2 dj - \bar{L^2}\right) \Big| \omega_{ki}\right]$$

- Solving this for a^1_{ki} and a^2_{ki}, we get that the best response of any type of an agent is given by

$$a^1_{ki} = (1-r)\mathbb{E}[\theta^1|\omega_{ki}] + r\mathbb{E}[\bar{a}^1|\omega_{ki}]$$
$$a^2_{ki} = (1-r)\mathbb{E}[\theta^2|\omega_{ki}] + r\mathbb{E}[\bar{a}^2|\omega_{ki}]$$

Characterization: Transformed Public Signal

- An agent of type k infers information about fundamental θ^1 using the unbiased private signal x_{ki}^1 and the unbiased public signal which is now transformed to
$$y_k^1 = \frac{y - (1-\lambda)x_{ki}^2}{\lambda}$$
- An agent of type k infers information about fundamental θ^2 using the unbiased private signal x_{ki}^2 and the unbiased public signal which is now transformed to
$$y_k^2 = \frac{y - \lambda x_{ki}^1}{1-\lambda}$$

Characterization: Bayesian Weights

- Define the Bayesian weights of the transformed public signal for the expected values of corresponding fundamentals in posterior distributions

$$B_1^1 = \frac{\left(\frac{(1-\lambda)^2}{\lambda^2}(\kappa_{1x}^2)^{-1} + \frac{1}{\lambda^2}(\kappa_y)^{-1}\right)^{-1}}{\left(\frac{(1-\lambda)^2}{\lambda^2}(\kappa_{1x}^2)^{-1} + \frac{1}{\lambda^2}(\kappa_y)^{-1}\right)^{-1} + \kappa_{.x}^1}$$

$$B_2^1 = \frac{\left(\frac{(1-\lambda)^2}{\lambda^2}(\kappa_{2x}^2)^{-1} + \frac{1}{\lambda^2}(\kappa_y)^{-1}\right)^{-1}}{\left(\frac{(1-\lambda)^2}{\lambda^2}(\kappa_{2x}^2)^{-1} + \frac{1}{\lambda^2}(\kappa_y)^{-1}\right)^{-1} + \kappa_{.x}^1}$$

$$B_1^2 = \frac{\left(\frac{\lambda^2}{(1-\lambda)^2}(\kappa_{.x}^1)^{-1} + \frac{1}{(1-\lambda)^2}(\kappa_y)^{-1}\right)^{-1}}{\left(\frac{\lambda^2}{(1-\lambda)^2}(\kappa_{.x}^1)^{-1} + \frac{1}{(1-\lambda)^2}(\kappa_y)^{-1}\right)^{-1} + \kappa_{1x}^2}$$

$$B_2^2 = \frac{\left(\frac{\lambda^2}{(1-\lambda)^2}(\kappa_{.x}^1)^{-1} + \frac{1}{(1-\lambda)^2}(\kappa_y)^{-1}\right)^{-1}}{\left(\frac{\lambda^2}{(1-\lambda)^2}(\kappa_{.x}^1)^{-1} + \frac{1}{(1-\lambda)^2}(\kappa_y)^{-1}\right)^{-1} + \kappa_{2x}^2}$$

- Notice that

$$B_1^1 < B_2^1$$
$$B_1^2 > B_2^2$$

Posterior Distributions

$$\theta^1 \Big| \omega_{1i} \sim N\left(B_1^1 y_1^1 + (1 - B_1^1) x_{1i}^1, \frac{1}{\left((\kappa_{2x}^2)^{-1} + \frac{1}{\lambda^2}(\kappa_y)^{-1}\right)^{-1} + \kappa_{.x}^1}\right)$$

$$\theta^1 \Big| \omega_{2i} \sim N\left(B_2^1 y_2^1 + (1 - B_2^1) x_{2i}^1, \frac{1}{\left(\frac{(1-\lambda)^2}{\lambda^2}(\kappa_{2x}^2)^{-1} + \frac{1}{\lambda^2}(\kappa_y)^{-1}\right)^{-1} + \kappa_{.x}^1}\right)$$

$$\theta^2 \Big| \omega_{1i} \sim N\left(B_1^2 y_1^2 + (1 - B_1^2) x_{1i}^2, \frac{1}{\left(\frac{\lambda^2}{(1-\lambda)^2}(\kappa_{.x}^1)^{-1} + \frac{1}{(1-\lambda)^2}(\kappa_y)^{-1}\right)^{-1} + \kappa_{1x}^2}\right)$$

$$\theta^2 \Big| \omega_{2i} \sim N\left(B_2^2 y_2^2 + (1 - B_2^2) x_{2i}^2, \frac{1}{\left(\frac{\lambda^2}{(1-\lambda)^2}(\kappa_{.x}^1)^{-1} + \frac{1}{(1-\lambda)^2}(\kappa_y)^{-1}\right)^{-1} + \kappa_{2x}^2}\right)$$

Results

1. The equilibrium is given by the linear functions for each type $k \in \{1,2\}$:
$$a^1_{ki} = \psi^1_{kx} x^1_{ki} + \psi^2_{kx} x^2_{ki} + \psi^1_{ky} y$$
$$a^2_{ki} = \varphi^1_{kx} x^1_{ki} + \varphi^2_{kx} x^2_{ki} + \varphi^2_{ky} y$$
2. This linear equilibrium is the unique equilibrium
3. Type I agents are relatively more sensitive to the public signal when choosing their action about θ^2 compared to Type II agents, while Type II agents are relatively more sensitive to the public signal when choosing their action about θ^1 compared to Type I agents, i.e.
$$\varphi^2_{1y} > \varphi^2_{2y}, \quad \psi^1_{1y} < \psi^1_{2y}$$
Type I agents are relatively more sensitive to their private signals when choosing their action about θ^1 compared to Type II agents, while Type II agents are relatively more sensitive to their private signals when choosing their action about θ^2 compared to Type I agents, i.e.
$$\psi^1_{1x} > \psi^1_{2x}, \quad \varphi^2_{1x} < \varphi^2_{2x}$$

Math: Coefficients on *xs*

$$\psi^1_{1x} = \frac{B^1_1 r - r - B^1_1 + B^2_2 r - B^1_1 B^2_2 r + B^2_1 B^1_2 r + B^2_1 \lambda r - B^2_2 \lambda r + B^1_1 B^2_2 \lambda r - B^2_1 B^1_2 \lambda r + 1}{B^1_2 r - r + B^2_2 r + B^1_1 \lambda r + B^2_1 \lambda r - B^1_2 \lambda r - B^2_2 \lambda r + 1}$$

$$\psi^2_{1x} = -\frac{(1-\lambda)\left(B^1_1 - B^1_1 r + B^1_2 r + B^1_1 B^2_2 r - B^2_1 B^1_2 r + B^1_1 \lambda r - B^1_2 \lambda r - B^1_1 B^2_2 \lambda r + B^2_1 B^1_2 \lambda r\right)}{\lambda\left(B^1_2 r - r + B^2_2 r + B^1_1 \lambda r + B^2_1 \lambda r - B^1_2 \lambda r - B^2_2 \lambda r + 1\right)}$$

$$\psi^1_{2x} = \frac{B^1_2 r - r - B^1_2 + B^2_2 r + B^2_1 \lambda r - B^2_2 \lambda r + B^1_1 B^2_2 \lambda r - B^2_1 B^1_2 \lambda r + 1}{B^1_2 r - r + B^2_2 r + B^1_1 \lambda r + B^2_1 \lambda r - B^1_2 \lambda r - B^2_2 \lambda r + 1}$$

$$\psi^2_{2x} = -\frac{(1-\lambda)(B^1_2 + B^1_1 \lambda r - B^1_2 \lambda r - B^1_1 B^2_2 \lambda r + B^2_1 B^1_2 \lambda r)}{\lambda(B^1_2 r - r + B^2_2 r + B^1_1 \lambda r + B^2_1 \lambda r - B^1_2 \lambda r - B^2_2 \lambda r + 1)}$$

$$\varphi^1_{1x} = -\frac{\lambda\left(B^2_1 - B^2_1 r + B^2_2 r + B^2_1 B^1_2 r - B^1_1 B^2_2 r + B^2_1 \lambda r - B^2_2 \lambda r - B^2_1 B^1_2 \lambda r + B^1_1 B^2_2 \lambda r\right)}{(1-\lambda)\left(B^1_2 r - r + B^2_2 r + B^1_1 \lambda r + B^2_1 \lambda r - B^1_2 \lambda r - B^2_2 \lambda r + 1\right)}$$

$$\varphi^2_{1x} = \frac{B^2_1 r - r - B^2_1 + B^1_2 r - B^2_1 B^1_2 r + B^1_1 B^2_2 r + B^1_1 \lambda r - B^1_2 \lambda r + B^2_1 B^1_2 \lambda r - B^1_1 B^2_2 \lambda r + 1}{B^1_2 r - r + B^2_2 r + B^1_1 \lambda r + B^2_1 \lambda r - B^1_2 \lambda r - B^2_2 \lambda r + 1}$$

$$\varphi^1_{2x} = -\frac{\lambda(B^2_2 + B^2_1 \lambda r - B^2_2 \lambda r - B^2_1 B^1_2 \lambda r + B^1_1 B^2_2 \lambda r)}{(1-\lambda)(B^1_2 r - r + B^2_2 r + B^1_1 \lambda r + B^2_1 \lambda r - B^1_2 \lambda r - B^2_2 \lambda r + 1)}$$

$$\varphi^2_{2x} = \frac{B^2_2 r - r - B^2_2 + B^1_2 r + B^1_1 \lambda r - B^1_2 \lambda r + B^2_1 B^1_2 \lambda r - B^1_1 B^2_2 \lambda r + 1}{B^1_2 r - r + B^2_2 r + B^1_1 \lambda r + B^2_1 \lambda r - B^1_2 \lambda r - B^2_2 \lambda r + 1}$$

Math: Coefficients on *ys*

$$\psi^1_{1y} = \frac{B^1_1 - B^1_1 r + B^1_2 r + B^1_1 B^2_2 r - B^2_1 B^1_2 r + B^1_1 \lambda r - B^1_2 \lambda r - B^1_1 B^2_2 \lambda r + B^2_1 B^1_2 \lambda r}{\lambda(B^1_2 r - r + B^2_2 r + B^1_1 \lambda r + B^2_1 \lambda r - B^1_2 \lambda r - B^2_2 \lambda r + 1)}$$

$$\psi^1_{2y} = \frac{B^1_2 + B^1_1 \lambda r - B^1_2 \lambda r - B^1_1 B^2_2 \lambda r + B^2_1 B^1_2 \lambda r}{\lambda(B^1_2 r - r + B^2_2 r + B^1_1 \lambda r + B^2_1 \lambda r - B^1_2 \lambda r - B^2_2 \lambda r + 1)}$$

$$\varphi^2_{1y} = \frac{B^2_1 - B^2_1 r + B^2_2 r + B^2_1 B^1_2 r - B^1_1 B^2_2 r + B^2_1 \lambda r - B^2_2 \lambda r - B^2_1 B^1_2 \lambda r + B^1_1 B^2_2 \lambda r}{(1-\lambda)(B^1_2 r - r + B^2_2 r + B^1_1 \lambda r + B^2_1 \lambda r - B^1_2 \lambda r - B^2_2 \lambda r + 1)}$$

$$\varphi^2_{2y} = \frac{B^2_2 + B^2_1 \lambda r - B^2_2 \lambda r - B^2_1 B^1_2 \lambda r + B^1_1 B^2_2 \lambda r}{(1-\lambda)(B^1_2 r - r + B^2_2 r + B^1_1 \lambda r + B^2_1 \lambda r - B^1_2 \lambda r - B^2_2 \lambda r + 1)}$$

Social Welfare Function

- Define the (normalized) social welfare function

$$W(\mathbf{a},\boldsymbol{\theta}) \equiv \frac{1}{1-r}\int u_i(\mathbf{a},\boldsymbol{\theta})di = -\lambda\int(a_{1i}^1-\theta^1)^2di - \lambda\int(a_{1i}^2-\theta^2)^2di - (1-\lambda)\int(a_{2i}^1-\theta^1)^2dj - (1-\lambda)\int(a_{2i}^2-\theta^2)^2dj$$

- Rewrite the equilibrium action as

$$a_{ki}^1 = \theta^1 + \psi_{kx}^1\nu_{.i}^1 + \psi_{kx}^2\nu_{ki}^2 + \psi_{ky}^1\varepsilon$$
$$a_{ki}^2 = \theta^2 + \varphi_{kx}^1\nu_{.i}^1 + \varphi_{kx}^2\nu_{ki}^2 + \varphi_{ky}^2\varepsilon$$

- Expected welfare is given:

$$\mathbb{E}[W|\boldsymbol{\theta}] = -\Bigg[\left(\lambda\left((\psi_{1x}^1)^2+(\varphi_{1x}^1)^2\right)+(1-\lambda)\left((\psi_{2x}^1)^2+(\varphi_{2x}^1)^2\right)\right)\frac{1}{\kappa_{.x}^1} + \left(\lambda\left((\psi_{1y}^1)^2+(\varphi_{1y}^1)^2\right)+(1-\lambda)\left((\psi_{2y}^1)^2+(\varphi_{2y}^1)^2\right)\right)\frac{1}{\kappa_y} + \left(\lambda\left((\psi_{1x}^2)^2+(\varphi_{1x}^2)^2\right)\frac{1}{(\kappa_{1x}^2)}+(1-\lambda)\left((\psi_{2x}^2)^2+(\varphi_{2x}^2)^2\right)\frac{1}{(\kappa_{2x}^2)}\right)\Bigg]$$

Preliminaries for Quantitative Results

- Case when $\kappa^1_{.x} = \kappa^2_{2x}$
- Define $\kappa_x = \kappa^1_{.x} = \kappa^2_{2x}$ and normalize it to 1
- Define $\delta = \frac{\kappa^2_{1x}}{\kappa^2_{2x}} < 1$ – the inverse relative dispersion of the private signals about θ^2 between two types of agents

We want to see how results of Morris and Shin (2002) are affected by different values of the parameters δ and λ (in their model $\delta = 1$, $\lambda = 1$)

- **M&S** benchmark: $\frac{\partial \mathbb{E}[W|\boldsymbol{\theta}]}{\partial \kappa_x} = 0$ when $\kappa_y = (2r-1)(1-r)\kappa_x$
- In my framework I found that for $\lambda = 0.5$, $\delta = 1$ the new benchmark is now **M&S***: $\frac{\partial \mathbb{E}[W|\boldsymbol{\theta}]}{\partial \kappa_x} = 0$ when $\kappa_y = \mathbf{2}(2r-1)(1-r)\kappa_x$
- Define $\tau = \frac{\kappa_y}{2(2r-1)(1-r)\kappa_x}$ and normalize it to 1 for $\delta = 1$ and $\lambda = 0.5$

Indifference Curve

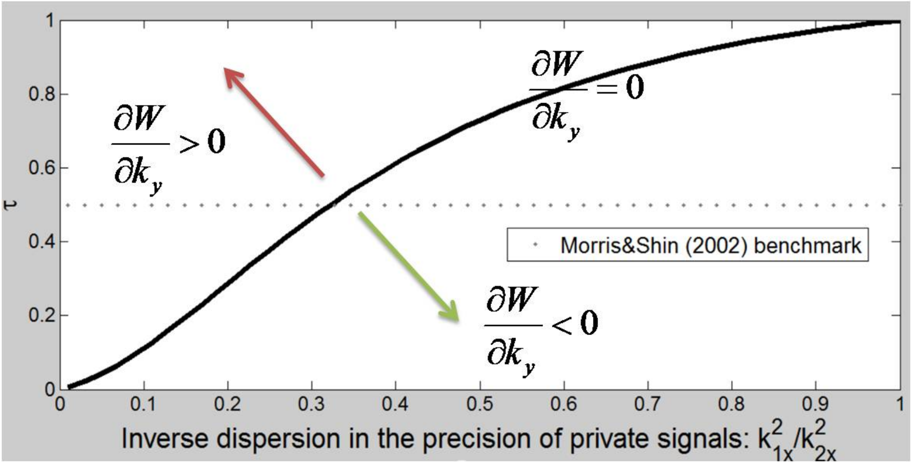
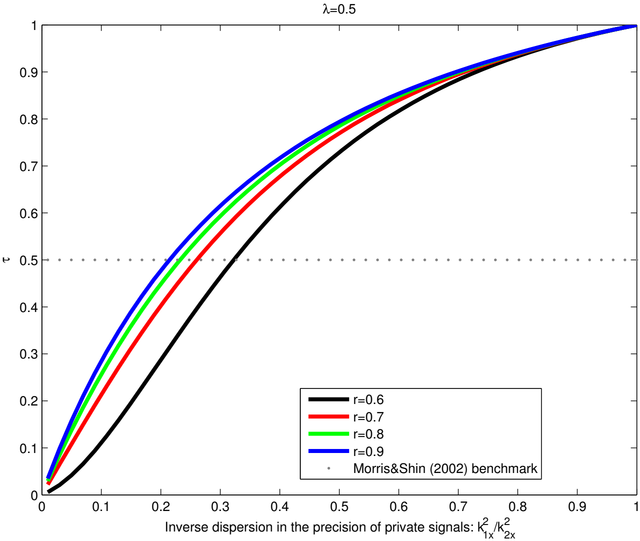
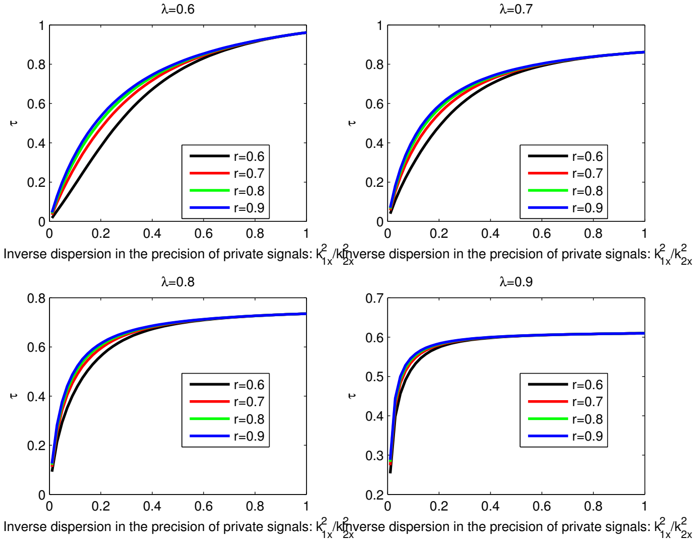


Figure: $\frac{\partial W}{\partial k_y} = 0, \lambda = 0.5, r = 0.6$

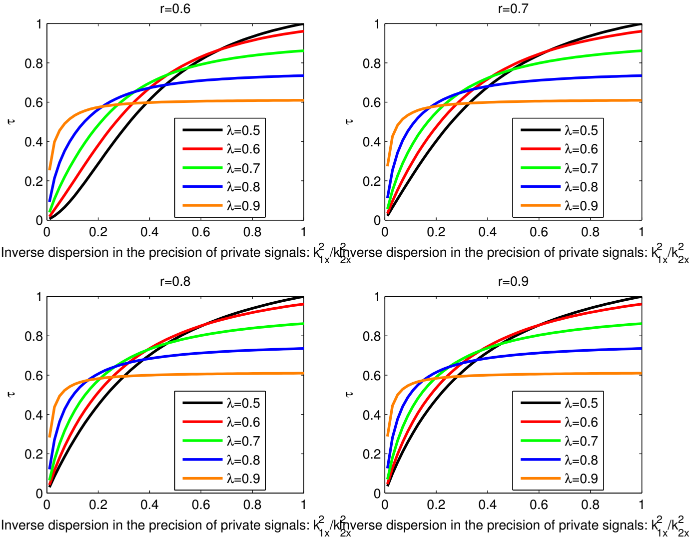
Effects of r on Welfare for $\lambda = 0.5$



Effects of r on Welfare for Different λ



Effects of λ on Welfare for Different r



Summary of Theoretical Results

- Relative dispersion of private signals (asymmetry) matters in determining the social value of private information
  - Different from Morris and Shin (2002, AER) that private signals be not very precise
- Requirements for the initial value precision of the public information to be beneficial to welfare are milder when the dispersion is higher
  - Can even be milder compared to Morris and Shin (2002, AER)
- An increase in *r* makes agents more concerned about the precision of the public signal to have beneficiary effects on welfare
- An increase in λ increases the range where relatively low dispersion ($\delta > \delta^*$) does not play a role. At the same time, the results become more sensitive in case of relatively high dispersion ($\delta < \delta^*$)
- Evidence that the share of information content about fundamentals in the public signal, λ, is more important for welfare than strategic complementarity parameter, *r*

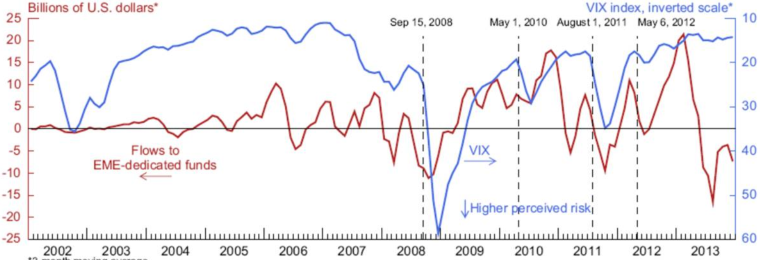
Discussion

- There are no micro-foundations but assuming that capital flows decisions are positively related to choices of actions about the fundamentals in my model, my results for capital flows are as follows
  - Rise in the precision of VIX are beneficial for social welfare in cases of Zambia and Mozambique – greater initial informational barriers without the public signal
  - Rise in the precision of VIX can hurt social welfare in cases of Chile and other countries with relatively lower dispersion of private signals
- The framework of my model is relatively rich enough to study welfare implications for two types of agents separately
- Next step is to add micro-foundations: Angeletos and La'o (2009), La'o (2010) develop models with dispersed information, uncertainty about current aggregate economic activity and learning and then merge them into RBC framework
  - Possibility of adding financial frictions (differences in the financial development)
  - Relate to Capital Flows

Conclusions

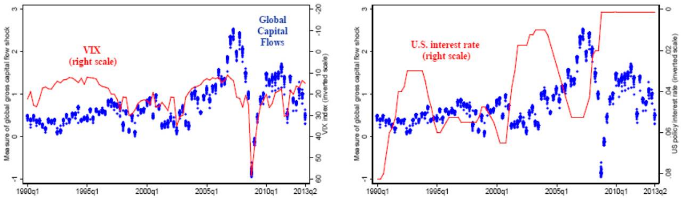
1. Motivated by the empirical evidence that there is a common component of capital flows related to VIX
2. Look at this evidence from a different angle with the intention to apply information theories
3. Contribute to the theoretical literature on the social value of the public information
4. Get results and relate them to welfare implications for capital flows

Appendix: Capital Flows and VIX



*3-month moving average.
The dates mark the following events: Sept 15, 2008, Lehman bankruptcy; May 1, 2010, increased concerns over Greece; August 1, 2011, increased concerns over Italy and Spain; May 6, 2012, increased concerns over Spanish banks, outcome of Greek election.

Country-specific Measure of Global Capital Flows, VIX and US Interest Rate



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Proposal Defense

Supplementary Material: Model

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Model

I follow Morris and Shin (2002) to build the model. It is based on a game with continuous actions, strategic complementarity, and dispersed information that induces strategic behavior in the spirit of the "beauty contest" example mentioned in Keynes's General Theory (1936).

I extend their model by adding a second type of agents with a different precision of a private signal. Now each type of agents chooses two actions related to two various fundamentals rather than one action as in Morris and Shin (2002). The precision of private signals about one fundamental is asymmetric across two types of agents, while it is symmetric for another fundamental. I also change the role of a public signal which now carries information about two fundamentals.

Setup

There is a continuum of agents indexed by $i \in [0,1]$ of two types: $\lambda > 0.5$ measure of type I (view it as a share of advanced economies) and $1-\lambda$ measure of type II (view it as a share of EMEs). Each type $k \in \{1,2\}$ of agents chooses two types of actions $a^1_{ki} \in \mathbb{R}$ and $a^2_{ki} \in \mathbb{R}$ to maximize expected payoffs.

$$U_{ki}(a_{ki},\mathbf{a},\theta) = -(1-r)(a^1_{ki}-\theta^1)^2 - r(L^1_{ki}-\bar{L}^1) - (1-r)(a^2_{ki}-\theta^2)^2 - r(L^2_{ki}-\bar{L}^2)$$

where $k \in \{1,2\}$ denotes the type of the agent i and the superscript denotes fundamentals (1 stands for θ^1 and 2 stands for θ^2), $\mathbf{a}$ is the action profile over all agents, θ^1 and θ^2 are two fundamentals, $r \in [-1;1]$ governs the strategic interaction between agents and for $h \in \{1,2\}$

$$L^h_{ki} \equiv \int (a^h_{ki} - a^h_{.j})^2 dj$$
$$\bar{L}^h \equiv \int L^h_{.j} dj$$

The timing is as follows. First, Nature draws θ^1 and θ^2 from improper uniform priors over the real line. Agents receive two types of exogenous private signals:

$$x^1_{.i} = \theta^1 + \nu^1_{.i} \quad \text{with} \quad \nu^1_{.i} \sim N\left(0, \left(\kappa^1_{.x}\right)^{-1}\right)$$
$$x^2_{ki} = \theta^2 + \nu^2_{ki} \quad \text{with} \quad \nu^2_{ki} \sim N\left(0, \left(\kappa^2_{kx}\right)^{-1}\right)$$

where $k \in \{1,2\}$ denotes the type of the agent i and the superscript denotes fundamentals (1 stands for θ^1 and 2 stands for θ^2). Assume $\kappa^1_{.x} > \kappa^2_{2x} > \kappa^2_{1x}$. So the private signal about θ^2 is more precise for agents of type II, while the private signal about θ^1 is equally precise for both types of agents and it is more precise than the any private signal about θ^2. This captures the asymmetry of private information in revealing the state in the economy of type II agents (similar to a home bias but it is for information here). In contrast, private information is symmetric concerning the fundamental in the economy of type I agents and it is more precise than private information of the economy of type II agents capturing the notion that there is less uncertainty in the private signals in advanced economies.

At the same time, agents receive an exogenous public signal which is a weighted average of two fundamentals:

$$y = \lambda\theta^1 + (1-\lambda)\theta^2 + \varepsilon \quad \text{with} \quad \varepsilon \sim N\left(0, (\kappa_y)^{-1}\right)$$

Once agents observe their signals, agents choose their actions and payoffs are realized. For this game we find a Bayesian Nash equilibrium.

Agent's Best Response Function

Each type of agents $k \in \{1,2\}$ solves

$$\max_{a_{ki}^1, a_{ki}^2} \mathbb{E}\left[-(1-r)(a_{ki}^1 - \theta^1)^2 - r\left(\int (a_{ki}^1 - a_{\cdot j}^1)^2 dj - \bar{L}^1\right) - \right.$$
$$\left. (1-r)(a_{ki}^2 - \theta^2)^2 - r\left(\int (a_{ki}^2 - a_{\cdot j}^2)^2 dj - \bar{L}^2\right) \middle| \omega_{ki}\right]$$

Solving this for a_{ki}^1 and a_{ki}^2, we get that the best response of any type of an agent is given by

$$a_{ki}^1 = (1-r)\mathbb{E}[\theta^1|\omega_{ki}] + r\mathbb{E}[\bar{a}^1|\omega_{ki}]$$
$$a_{ki}^2 = (1-r)\mathbb{E}[\theta^2|\omega_{ki}] + r\mathbb{E}[\bar{a}^2|\omega_{ki}]$$

Characterization

Each type of agents uses both signals to get evidence about the true fundamental. It is useful to rewrite the public signal in the following way:

$$\frac{y}{\lambda} = \theta^1 + \frac{1-\lambda}{\lambda}\theta^2 + \frac{\varepsilon}{\lambda}$$
$$\frac{y}{1-\lambda} = \theta^2 + \frac{\lambda}{1-\lambda}\theta^1 + \frac{\varepsilon}{1-\lambda}$$

As an example, consider how an agent of type k infers information about fundamental θ^1. The agent uses the unbiased private signal x_{ki}^1 and the unbiased public signal which is now transformed to

$$y_k^1 = \frac{y - (1-\lambda)x_{ki}^2}{\lambda}$$

where I use that θ^2 is inferred from $x_{ki}^2 - \nu_{ki}^2$.

Similarly, an agent of type k infers information about fundamental θ^2 from the unbiased private signal x_{ki}^2 and the unbiased public signal which is now transformed to

$$y_k^2 = \frac{y - \lambda x_{ki}^1}{1-\lambda}$$

where I use that θ^1 is inferred from $x_{ki}^1 - \nu_{ki}^1$.

Define

$$B_1^1 = \frac{\left(\frac{(1-\lambda)^2}{\lambda^2}(\kappa_{1x}^2)^{-1} + \frac{1}{\lambda^2}(\kappa_y)^{-1}\right)^{-1}}{\left(\frac{(1-\lambda)^2}{\lambda^2}(\kappa_{1x}^2)^{-1} + \frac{1}{\lambda^2}(\kappa_y)^{-1}\right)^{-1} + \kappa_{.x}^1}$$

$$B_2^1 = \frac{\left(\frac{(1-\lambda)^2}{\lambda^2}(\kappa_{2x}^2)^{-1} + \frac{1}{\lambda^2}(\kappa_y)^{-1}\right)^{-1}}{\left(\frac{(1-\lambda)^2}{\lambda^2}(\kappa_{2x}^2)^{-1} + \frac{1}{\lambda^2}(\kappa_y)^{-1}\right)^{-1} + \kappa_{.x}^1}$$

$$B_1^2 = \frac{\left(\frac{\lambda^2}{(1-\lambda)^2}(\kappa_{.x}^1)^{-1} + \frac{1}{(1-\lambda)^2}(\kappa_y)^{-1}\right)^{-1}}{\left(\frac{\lambda^2}{(1-\lambda)^2}(\kappa_{.x}^1)^{-1} + \frac{1}{(1-\lambda)^2}(\kappa_y)^{-1}\right)^{-1} + \kappa_{1x}^2}$$

$$B_2^2 = \frac{\left(\frac{\lambda^2}{(1-\lambda)^2}(\kappa_{.x}^1)^{-1} + \frac{1}{(1-\lambda)^2}(\kappa_y)^{-1}\right)^{-1}}{\left(\frac{\lambda^2}{(1-\lambda)^2}(\kappa_{.x}^1)^{-1} + \frac{1}{(1-\lambda)^2}(\kappa_y)^{-1}\right)^{-1} + \kappa_{2x}^2}$$

These variables define the Bayesian weights of the transformed public signal for the expected values of corresponding fundamentals in posterior distributions. Notice that

$$B_1^1 < B_2^1$$
$$B_1^2 > B_2^2$$

Then, based on both private and public information, the distribution of fundamentals given the information of agent i of type $k \in \{1,2\}$ is:

$$\theta^1 \Big| \omega_{1i} \sim N\left(B_1^1 y_1^1 + (1 - B_1^1)x_{1i}^1, \frac{1}{\left((\kappa_{2x}^2)^{-1} + \frac{1}{\lambda^2}(\kappa_y)^{-1}\right)^{-1} + \kappa_{.x}^1}\right)$$

$$\theta^1 \Big| \omega_{2i} \sim N\left(B_2^1 y_2^1 + (1 - B_2^1)x_{2i}^1, \frac{1}{\left(\frac{(1-\lambda)^2}{\lambda^2}(\kappa_{2x}^2)^{-1} + \frac{1}{\lambda^2}(\kappa_y)^{-1}\right)^{-1} + \kappa_{.x}^1}\right)$$

$$\theta^2 \Big| \omega_{1i} \sim N\left(B_1^2 y_1^2 + (1 - B_1^2)x_{1i}^2, \frac{1}{\left(\frac{\lambda^2}{(1-\lambda)^2}(\kappa_{.x}^1)^{-1} + \frac{1}{(1-\lambda)^2}(\kappa_y)^{-1}\right)^{-1} + \kappa_{1x}^2}\right)$$

$$\theta^2 \Big| \omega_{2i} \sim N\left(B_2^2 y_2^2 + (1 - B_2^2)x_{2i}^2, \frac{1}{\left(\frac{\lambda^2}{(1-\lambda)^2}(\kappa_{.x}^1)^{-1} + \frac{1}{(1-\lambda)^2}(\kappa_y)^{-1}\right)^{-1} + \kappa_{2x}^2}\right)$$

Result 1: The equilibrium is given by the linear functions for each type $k \in \{1,2\}$:

$$a_{ki}^1 = \psi_{kx}^1 x_{ki}^1 + \psi_{kx}^2 x_{ki}^2 + \psi_{ky}^1 y$$
$$a_{ki}^2 = \varphi_{kx}^1 x_{ki}^1 + \varphi_{kx}^2 x_{ki}^2 + \varphi_{ky}^2 y$$

where coefficients on xs equal

$$\psi^1_{1x} = \frac{B^1_1 r - r - B^1_1 + B^2_2 r - B^1_1 B^2_2 r + B^2_1 B^1_2 r + B^2_1\lambda r - B^2_2\lambda r + B^1_1 B^2_2\lambda r - B^2_1 B^1_2\lambda r + 1}{B^1_2 r - r + B^2_2 r + B^1_1\lambda r + B^2_1\lambda r - B^1_2\lambda r - B^2_2\lambda r + 1}$$
$$\psi^2_{1x} = -\frac{(1-\lambda)\left(B^1_1 - B^1_1 r + B^1_2 r + B^1_1 B^2_2 r - B^2_1 B^1_2 r + B^1_1\lambda r - B^1_2\lambda r - B^1_1 B^2_2\lambda r + B^2_1 B^1_2\lambda r\right)}{\lambda\left(B^1_2 r - r + B^2_2 r + B^1_1\lambda r + B^2_1\lambda r - B^1_2\lambda r - B^2_2\lambda r + 1\right)}$$
$$\psi^1_{2x} = \frac{B^1_2 r - r - B^1_2 + B^2_2 r + B^2_1\lambda r - B^2_2\lambda r + B^1_1 B^2_2\lambda r - B^2_1 B^1_2\lambda r + 1}{B^1_2 r - r + B^2_2 r + B^1_1\lambda r + B^2_1\lambda r - B^1_2\lambda r - B^2_2\lambda r + 1}$$
$$\psi^2_{2x} = -\frac{(1-\lambda)(B^1_2 + B^1_1\lambda r - B^1_2\lambda r - B^1_1 B^2_2\lambda r + B^2_1 B^1_2\lambda r)}{\lambda(B^1_2 r - r + B^2_2 r + B^1_1\lambda r + B^2_1\lambda r - B^1_2\lambda r - B^2_2\lambda r + 1)}$$
$$\varphi^1_{1x} = -\frac{\lambda\left(B^2_1 - B^2_1 r + B^2_2 r + B^2_1 B^1_2 r - B^1_1 B^2_2 r + B^2_1\lambda r - B^2_2\lambda r - B^2_1 B^1_2\lambda r + B^1_1 B^2_2\lambda r\right)}{(1-\lambda)\left(B^1_2 r - r + B^2_2 r + B^1_1\lambda r + B^2_1\lambda r - B^1_2\lambda r - B^2_2\lambda r + 1\right)}$$
$$\varphi^2_{1x} = \frac{B^2_1 r - r - B^2_1 + B^1_2 r - B^2_1 B^1_2 r + B^1_1 B^2_2 r + B^1_1\lambda r - B^1_2\lambda r + B^2_1 B^1_2\lambda r - B^1_1 B^2_2\lambda r + 1}{B^1_2 r - r + B^2_2 r + B^1_1\lambda r + B^2_1\lambda r - B^1_2\lambda r - B^2_2\lambda r + 1}$$
$$\varphi^1_{2x} = -\frac{\lambda(B^2_2 + B^2_1\lambda r - B^2_2\lambda r - B^2_1 B^1_2\lambda r + B^1_1 B^2_2\lambda r)}{(1-\lambda)(B^1_2 r - r + B^2_2 r + B^1_1\lambda r + B^2_1\lambda r - B^1_2\lambda r - B^2_2\lambda r + 1)}$$
$$\varphi^2_{2x} = \frac{B^2_2 r - r - B^2_2 + B^1_2 r + B^1_1\lambda r - B^1_2\lambda r + B^2_1 B^1_2\lambda r - B^1_1 B^2_2\lambda r + 1}{B^1_2 r - r + B^2_2 r + B^1_1\lambda r + B^2_1\lambda r - B^1_2\lambda r - B^2_2\lambda r + 1}$$

and where coefficients on ys equal

$$\psi^1_{1y} = \frac{B^1_1 - B^1_1 r + B^1_2 r + B^1_1 B^2_2 r - B^2_1 B^1_2 r + B^1_1\lambda r - B^1_2\lambda r - B^1_1 B^2_2\lambda r + B^2_1 B^1_2\lambda r}{\lambda(B^1_2 r - r + B^2_2 r + B^1_1\lambda r + B^2_1\lambda r - B^1_2\lambda r - B^2_2\lambda r + 1)}$$
$$\psi^1_{2y} = \frac{B^1_2 + B^1_1\lambda r - B^1_2\lambda r - B^1_1 B^2_2\lambda r + B^2_1 B^1_2\lambda r}{\lambda(B^1_2 r - r + B^2_2 r + B^1_1\lambda r + B^2_1\lambda r - B^1_2\lambda r - B^2_2\lambda r + 1)}$$
$$\varphi^2_{1y} = \frac{B^2_1 - B^2_1 r + B^2_2 r + B^2_1 B^1_2 r - B^1_1 B^2_2 r + B^2_1\lambda r - B^2_2\lambda r - B^2_1 B^1_2\lambda r + B^1_1 B^2_2\lambda r}{(1-\lambda)(B^1_2 r - r + B^2_2 r + B^1_1\lambda r + B^2_1\lambda r - B^1_2\lambda r - B^2_2\lambda r + 1)}$$
$$\varphi^2_{2y} = \frac{B^2_2 + B^2_1\lambda r - B^2_2\lambda r - B^2_1 B^1_2\lambda r + B^1_1 B^2_2\lambda r}{(1-\lambda)(B^1_2 r - r + B^2_2 r + B^1_1\lambda r + B^2_1\lambda r - B^1_2\lambda r - B^2_2\lambda r + 1)}$$

This linear equilibrium is the unique equilibrium.

Proof:

Part 1: For linear solutions guess and verify.

Without loss of generality let's show the procedure for the actions concerning the fundamental θ^1. Analogous steps result in the solution for another fundamental.

Given our guess the aggregate action about θ^1 is

$$\bar{a}^1 = (\lambda\psi^1_{1x} + (1-\lambda)\psi^1_{2x})\theta^1 + (\lambda\psi^2_{1x} + (1-\lambda)\psi^2_{2x})\theta^2 + (\lambda\psi^1_{1y} + (1-\lambda)\psi^1_{2y})y$$

Substitute into the best response function of the agents of type I:

$$a^1_{1i} = (1-r)\mathbb{E}[\theta^1|\omega_{1i}] + r\mathbb{E}[\bar{a}^1|\omega_{1i}] = \\ \left(1 - r + r(\lambda\psi^1_{1x} + (1-\lambda)\psi^1_{2x})\right)\mathbb{E}[\theta^1|\omega_{1i}] + r(\lambda\psi^2_{1x} + (1-\lambda)\psi^2_{2x})\mathbb{E}[\theta^2|\omega_{1i}] + \\ ry(\lambda\psi^1_{1y} + (1-\lambda)\psi^1_{2y})$$

Substitute for the expected values from the posterior distributions:

$$a_{1i}^1 = \left(1 - r + r(\lambda\psi_{1x}^1 + (1-\lambda)\psi_{2x}^1)\right)\left(B_1^1\left(\frac{y-(1-\lambda)x_{1i}^2}{\lambda}\right) + (1-B_1^1)x_{1i}^1\right) + r(\lambda\psi_{1x}^2 + (1-\lambda)\psi_{2x}^2)\left(B_1^2\left(\frac{y-\lambda x_{1i}^1}{1-\lambda}\right) + (1-B_1^2)x_{1i}^2\right) + ry(\lambda\psi_{1y}^1 + (1-\lambda)\psi_{2y}^1)$$

Collecting terms observe that it is linear, i.e.

$$a_{1i}^1 = \underbrace{\left((1-r+r(\lambda\psi_{1x}^1+(1-\lambda)\psi_{2x}^1))(1-B_1^1) - r(\lambda\psi_{1x}^2+(1-\lambda)\psi_{2x}^2)B_1^2\frac{\lambda}{1-\lambda}\right)}_{\psi_{1x}^1} x_{1i}^1 + \underbrace{\left(r(\lambda\psi_{1x}^2+(1-\lambda)\psi_{2x}^2)(1-B_1^2) - (1-r+r(\lambda\psi_{1x}^1+(1-\lambda)\psi_{2x}^1))\frac{1-\lambda}{\lambda}B_1^1\right)}_{\psi_{1x}^2} x_{1i}^2 + \underbrace{\left((1-r+r(\lambda\psi_{1x}^1+(1-\lambda)\psi_{2x}^1))\frac{B_1^1}{\lambda} + r(\lambda\psi_{1x}^2+(1-\lambda)\psi_{2x}^2)\frac{B_1^2}{1-\lambda} + r(\lambda\psi_{1y}^1+(1-\lambda)\psi_{2y}^1)\right)}_{\psi_{1y}^1} y$$

Similarly, using the same steps find

$$a_{2i}^1 = \underbrace{\left((1-r+r(\lambda\psi_{1x}^1+(1-\lambda)\psi_{2x}^1))(1-B_2^1) - r(\lambda\psi_{1x}^2+(1-\lambda)\psi_{2x}^2)B_2^2\frac{\lambda}{1-\lambda}\right)}_{\psi_{2x}^1} x_{1i}^1 + \underbrace{\left(r(\lambda\psi_{1x}^2+(1-\lambda)\psi_{2x}^2)(1-B_2^2) - (1-r+r(\lambda\psi_{1x}^1+(1-\lambda)\psi_{2x}^1))\frac{1-\lambda}{\lambda}B_2^1\right)}_{\psi_{2x}^2} x_{1i}^2 + \underbrace{\left((1-r+r(\lambda\psi_{1x}^1+(1-\lambda)\psi_{2x}^1))\frac{B_2^1}{\lambda} + r(\lambda\psi_{1x}^2+(1-\lambda)\psi_{2x}^2)\frac{B_2^2}{1-\lambda} + r(\lambda\psi_{1y}^1+(1-\lambda)\psi_{2y}^1)\right)}_{\psi_{2y}^1} y$$

Therefore, in order to find $\psi_{1x}^1, \psi_{1x}^2, \psi_{2x}^1, \psi_{2x}^2$ solve the system of four linear equations analytically. I use MatLab for this calculation. Using these solutions plug them into the representation of ψ_{1y}^1, ψ_{2y}^1 and solve the system of two equations to find these coefficients using the method of undetermined coefficients.

The same steps give solutions for the actions about fundamental θ^2. □

Part 2: Uniqueness result follows from the linearity of the best response function and the linearity of the expected value of the posterior distribution in case of normal priors. Using the same steps as in Morris and Shin (2002):

$$a_{ki}^j = (1-r)\sum_{h=0}^{\infty} r^h E_i\left[\bar{E}^{(h)}\theta^j\right]$$

where $k \in \{1{,}2\}$ and $j \in \{1{,}2\}$

The expected values of θ based on both private and public signals are linear with the weights reflecting the relative precision of signals. Plug it into the last expression and verify... □

Result 2: Type I agents are relatively more sensitive to the public signal when choosing their action about θ^2 compared to Type II agents, while Type II agents are relatively more sensitive to the public signal when choosing their action about θ^1 compared to Type I agents, i.e.

$$\varphi_{1y}^2 > \varphi_{2y}^2, \quad \psi_{1y}^1 < \psi_{2y}^1$$

Type I agents are relatively more sensitive to their private signals when choosing their action about θ^1 compared to Type II agents, while Type II agents are relatively more sensitive to their private signals when choosing their action about θ^2 compared to Type I agents, i.e.

$$\psi_{1x}^1 > \psi_{2x}^1, \quad \varphi_{1x}^2 < \varphi_{2x}^2$$

Proof: Use the relations for $B_{\cdot}s$ and plug them into the equilibrium functions. □

Welfare

Let's define the (normalized) social welfare function as the weighted average of individual utilities where weights are given by the shares of type I and type II agents.

$$W(\mathbf{a},\boldsymbol{\theta}) \equiv \frac{1}{1-r}\int u_i(\mathbf{a},\boldsymbol{\theta})di = -\lambda\int(a_{1i}^1-\theta^1)^2di - \lambda\int(a_{1i}^2-\theta^2)^2di - (1-\lambda)\int(a_{2i}^1-\theta^1)^2dj - (1-\lambda)\int(a_{2i}^2-\theta^2)^2dj$$

We can rewrite the equilibrium action as

$$a_{ki}^1 = \psi_{kx}^1(\theta^1+\nu_{.i}^1) + \psi_{kx}^2(\theta^2+\nu_{ki}^2) + \psi_{ky}^1(\lambda\theta^1+(1-\lambda)\theta^2+\varepsilon)$$
$$a_{ki}^2 = \varphi_{kx}^1(\theta^1+\nu_{.i}^1) + \varphi_{kx}^2(\theta^2+\nu_{ki}^2) + \varphi_{ky}^2(\lambda\theta^1+(1-\lambda)\theta^2+\varepsilon)$$

Simplify using coefficient's relationships:

$$a_{ki}^1 = \theta^1 + \psi_{kx}^1\nu_{.i}^1 + \psi_{kx}^2\nu_{ki}^2 + \psi_{ky}^1\varepsilon$$
$$a_{ki}^2 = \theta^2 + \varphi_{kx}^1\nu_{.i}^1 + \varphi_{kx}^2\nu_{ki}^2 + \varphi_{ky}^2\varepsilon$$

Notice that the actions are unbiased with different variances for two types of agents even for fundamental θ^1 (about which there is a symmetric precision in private signals) because information sets are different.

Expected welfare is given:

$$\mathbb{E}[W|\boldsymbol{\theta}] = -\Bigg[\left(\lambda\left((\psi_{1x}^1)^2 + (\varphi_{1x}^1)^2\right) + (1-\lambda)\left((\psi_{2x}^1)^2 + (\varphi_{2x}^1)^2\right)\right)\frac{1}{\kappa_{.x}^1} + \left(\lambda\left((\psi_{1y}^1)^2 + (\varphi_{1y}^1)^2\right) + (1-\lambda)\left((\psi_{2y}^1)^2 + (\varphi_{2y}^1)^2\right)\right)\frac{1}{\kappa_y} + \left(\lambda\left((\psi_{1x}^2)^2 + (\varphi_{1x}^2)^2\right)\frac{1}{(\kappa_{1x}^2)} + (1-\lambda)\left((\psi_{2x}^2)^2 + (\varphi_{2x}^2)^2\right)\frac{1}{(\kappa_{2x}^2)}\right)\Bigg]$$

Let's do comparative statics with respect to the precision of private and public information.

One of the features of my model is that precision of private signals about one fundamental affects the value of the relative weight placed on the public signal about another fundamental. Together with the choice of two actions this fact allows us to get different implications from Morris and Shin (2002).

Two main results of Morris and Shin (2002) are as follows

Result 1: Welfare is always increasing in the precision of private signals.

Result 2: Public signals can be bad for welfare because agents overreact to the public signal and if the coordination motive is relatively high enough ($r > 0.5$), then agents can coordinate at the expense of choosing actions that are further away from the true state. Private signals must not be very precise in order for increased precision of public information to be beneficial. Formally, if $\kappa_y > (2r-1)(1-r)\kappa_x$, then $\frac{\partial \mathbb{E}[W|\theta]}{\partial \kappa_x} > 0$

Firstly, my model has different implications for Result 1 of Morris and Shin (2002) because an increased precision of the private signal about fundamental θ^1 results in smaller relative precision of the private signal about another fundamental θ^2. So relative dispersion of private signals for two types of agents should also be taken into account.

Secondly, Result 2 of Morris and Shin (2002) changes as well. Not only the strategic complementarity parameter r but also relative precision of private signals and λ are important in determining welfare effects of a change in the precision of the public signal. Intuitively, if the dispersion between κ_{1x}^2 and κ_{2x}^2 is relatively large enough, then an increase in κ_y can increase welfare because type I agents can benefit from the public signal when inferring θ^2. At the same time, if the share of information about θ^1 is relatively large enough in the public signal which is determined by λ, then public signal says relatively less about θ^2 and hence type I agents require relatively greater precision of the public signal when inferring θ^2.

Quantitative analysis and Results

We are interested in how asymmetry in the precision of private signals for two types of agents affects welfare implications of changes in the precision of the public signal. Assume that $\kappa_{.x}^1 = \kappa_{2x}^2$, call them κ_x and normalize them to 1. Let $\delta = \frac{\kappa_{1x}^2}{\kappa_{2x}^2} < 1$, so δ defines the inverse relative dispersion of the private signals about θ^2 between two types of agents.

We want to see how results of Morris and Shin (2002) are affected by different values of the parameters δ and λ (in their model $\delta = 1$, $\lambda = 1$). The benchmark is

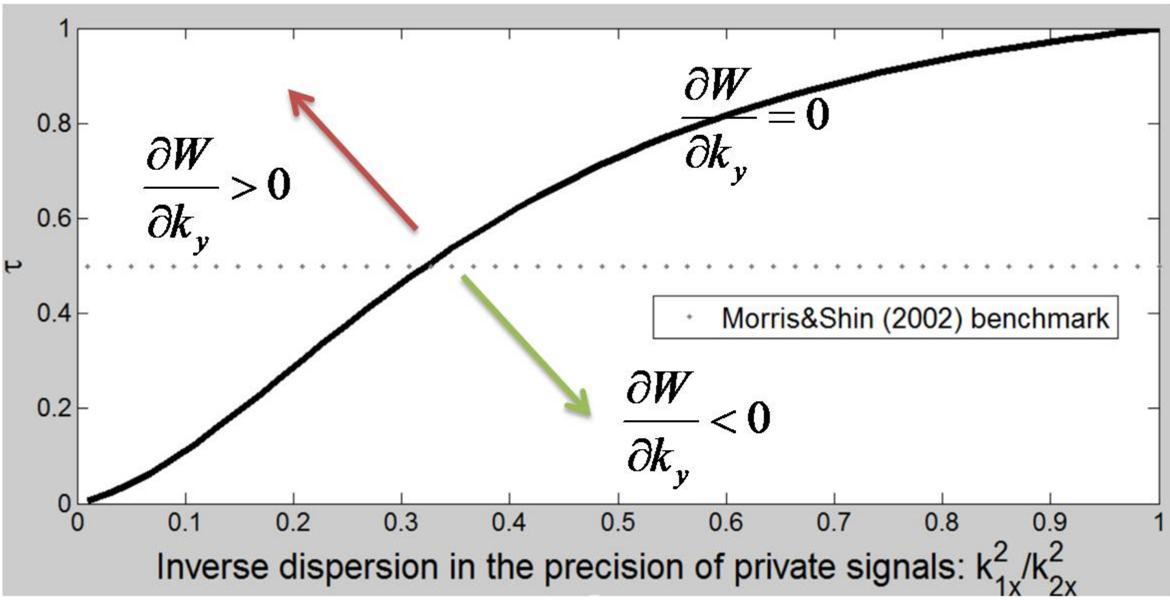

Figure 1: $\frac{\partial W}{\partial k_y} = 0, \lambda = 0.5, r = 0.6$

their equation that $\frac{\partial \mathbb{E}[W|\boldsymbol{\theta}]}{\partial \kappa_x} = 0$ when $\kappa_y = (2r-1)(1-r)\kappa_x$ for $r > 0.5$. Let's call it **M&S** benchmark.

In my framework I found that for $\lambda = 0.5$, $\delta = 1$ the new benchmark is now **M&S***: $\frac{\partial \mathbb{E}[W|\boldsymbol{\theta}]}{\partial \kappa_x} = 0$ when $\kappa_y = \mathbf{2}(2r-1)(1-r)\kappa_x$ for $\delta = 1$ and $r > 0.5$. The intuition behind this multiplication by 2 with no asymmetries of information is that now agents make decisions about 2 actions rather than 1 action as in Morris and Shin (2002).

Define $\tau = \frac{\kappa_y}{2(2r-1)(1-r)\kappa_x}$ and normalize it to 1 for $\delta = 1$. The interpretation of τ is that it shows the relative precision of the public signal with respect to the symmetric private signal, κ_x, such that there are no effects of changes in the precision of the public signal on welfare for a given dispersion parameter δ.

Figure 1 illustrates this concept of indifference (no effects of the precision of the public signal on welfare) graphically when $\lambda = 0.5, r = 0.6$. The points above this curve shows the combinations of δ and τ where an increase in the precision of the public signal increases welfare. The points above this curve shows the combinations of δ and τ where an increase in the precision of the public signal decreases welfare.

Note that Morris and Shin (2002) benchmark is when $\tau = 0.5$. The indifference curve intersects with $\tau = 0.5$ when the dispersion is around 0.35 meaning that if the inverse relative precision in private signals is less than 0.35, then an increase in the precision of the public signal increases welfare for the same value of the precision κ_x as in Morris and Shin (2002) where the effect on welfare is opposite. So in my framework we have more room for public signals to be beneficial on welfare in case of greater dispersion in the private signals.

Figure 2 illustrates the effects of changes in the complementarity parameter, r, on the indifference curve. An increase in r shifts the indifference curves to the left meaning that agents require more precise public signal to have beneficiary effects on welfare in case of greater complementarity. Intuitively, this is because they want to be more certain that they do not coordinate on noisy information.

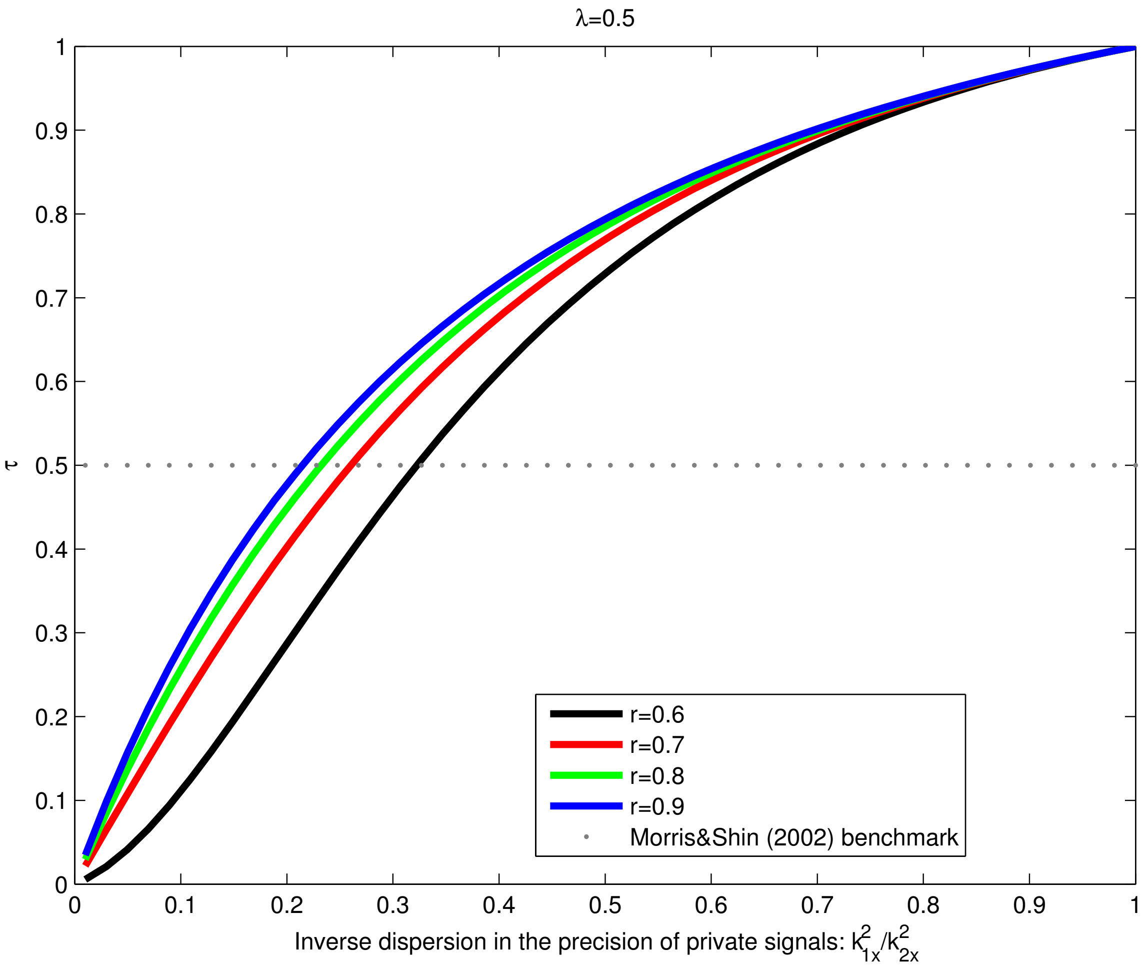

Figure 2: Effects of r on welfare for $\lambda = 0.5$

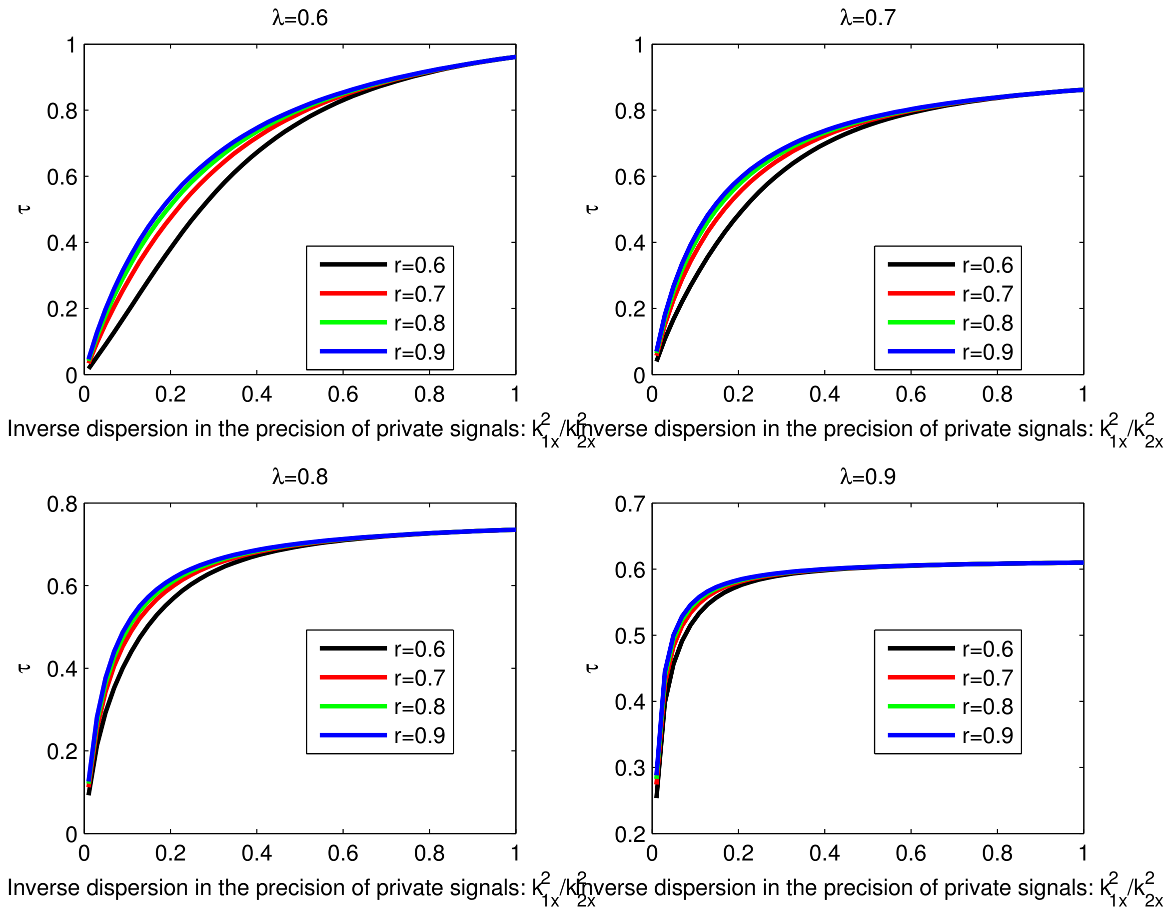

Figure 3: Effects of r on welfare for different λ

Figure 3 shows the effects of complementarity of actions on welfare for different values of the share of information about θ^1 in the public signal. We still have the same result about the effects of complementarity of actions on the indifference curves but now with greater λ the indifference curves for different values of r become closer to each other meaning that complementarity of actions becomes less important when the share of information about θ^1 in the public signal is greater.

Moreover, the curvatures of the indifference curves rise with greater λ. Intuitively, when λ is relatively large an increase in the dispersion parameter (up to some point) is considered to be less important in inferring θ^2 for type I agents because the information share of the public signal about θ^2 is small and so they cannot rely on it. Interestingly, even if the dispersion is very high (δ is near zero) an increase in the precision of the public signal reduces welfare for high λ and for low initial κ_y.

Figure 4 shows the effects of the share of information about θ^1 in the public signal on welfare for different values of complementarity of actions parameter. Changes in r do not affect the relative shapes and distances between the curves. It gives additional and even more clear evidence that λ are more significant than r in driving the results. Furthermore, for high λ there is a cutoff point for relative dispersion of the private signals that affects welfare implications considerably. It is around $\delta^* = 0.15$ for $r = 0.6$. In fact, the indifference curve is relatively flat for $\delta > \delta^*$ meaning that the dispersion in

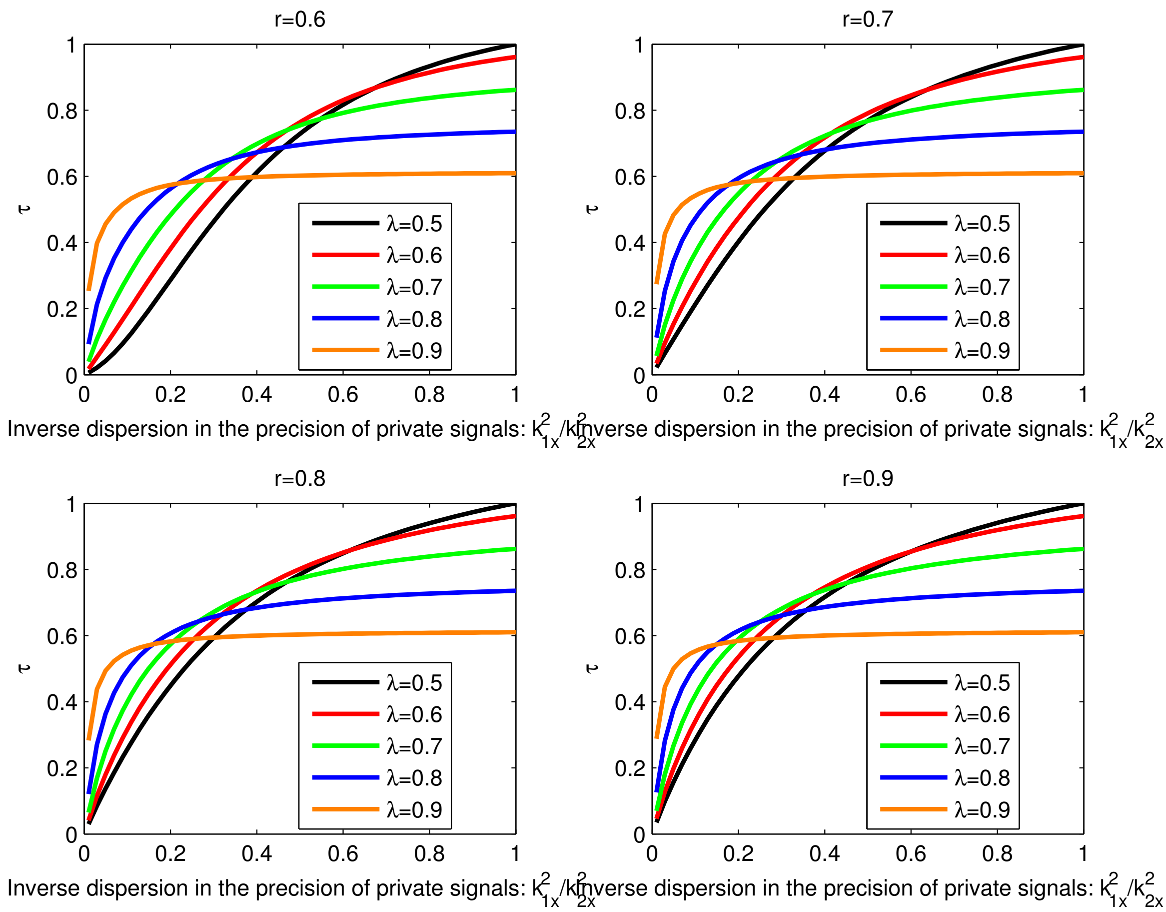

Figure 4: Effects of λ on welfare for different r

the precision does not matter much in this region, while the indifference curve is steep for $\delta < \delta^*$ meaning that we need to have a relatively precise public signal initially for positive welfare effects of public information. This cutoff point is less obvious for lower λ but it is present for different values of complementarity of actions.

MatLab Codes

Solving for coefficients

```
clear all

syms B11 B12 B21 B22 lambda r

% Solving for \psi %

A=[1-(1-B11)*r*lambda -(1-B11)*r*(1-lambda) B12*r*((lambda)^2)/(1-lambda) B12*r*lambda;
B11*(1-lambda)*r B11*r*((1-lambda)^2)/(lambda) (1-(1-B12)*r*lambda) -(1-B12)*(1-lambda)*r;
-(1-B21)*r*lambda 1-(1-B21)*r*(1-lambda) B22*r*((lambda)^2)/(1-lambda) B22*r*lambda;
B21*r*(1-lambda) B21*r*((1-lambda)^2)/(lambda) -(1-B22)*r*lambda 1-r*(1-lambda)*(1-B22)];

B=[(1-r)*(1-B11); -B11*(1-lambda)*(1-r)/lambda; (1-r)*(1-B21); -B21*(1-lambda)*(1-r)/lambda];

X = linsolve(A, B);  % 4 by 1 matrix with solutions for \psi coefficients on x: (\psi_{1x}^1, \psi_{2x}^1, \psi_{1x}^2, \psi_{2x}^2)' %

w1=lambda*X(1)+(1-lambda)*X(2);
w2=lambda*X(3)+(1-lambda)*X(4);

C=[1-r*lambda -(1-lambda)*r; -r*lambda 1-r*(1-lambda)];
D=[(1-r+r*w1)*B11/lambda+r*w2*B12/(1-lambda); (1-r+r*w1)*B21/lambda+r*w2*B22/(1-lambda)];

Y=linsolve(C, D);  % 2 by 1 matrix with solutions for \psi coefficients on y: (\psi_{1y}^1, \psi_{2y}^1)' %


% Solving for \phi %

A2=[1-(1-B12)*r*lambda -(1-B12)*r*(1-lambda) B11*r*(1-lambda) B11*r*((1-lambda)^2)/(lambda);
B12*r*(lambda^2)/(1-lambda) B12*r*(lambda) (1-(1-B11)*r*lambda) -(1-B11)*(1-lambda)*r;
-(1-B22)*r*lambda 1-(1-B22)*r*(1-lambda) B21*r*(1-lambda) B21*r*((1-lambda)^2)/(lambda);
B22*r*(lambda^2)/(1-lambda) B22*r*(lambda) -(1-B21)*r*lambda 1-r*(1-lambda)*(1-B21)];

B2=[(1-r)*(1-B12); -B12*(lambda)*(1-r)/(1-lambda); (1-r)*(1-B22); -B22*(lambda)*(1-r)/(1-lambda)];

X2 = linsolve(A2, B2);  % 4 by 1 matrix with solutions for \phi coefficients on x: (\phi_{1x}^2, \phi_{2x}^2, \phi_{1x}^1, \phi_{2x}^2)' %

q1=lambda*X2(1)+(1-lambda)*X2(2);
```

```
q2=lambda*X2(3)+(1-lambda)*X2(4);

C2=[1-r*lambda -(1-lambda)*r; -r*lambda 1-r*(1-lambda)];
D2=[(1-r+r*q1)*B12/(1-lambda)+r*q2*B11/(lambda); (1-r+r*q1)*B22/(1-
lambda)+r*q2*B21/(lambda)];

Y2=linsolve(C2, D2);  % 2 by 1 matrix with solutions for \phi coefficients on y: (\phi_{1y}^2,
\phi_{2y}^2)' %
```

Wealth calculation

```
syms k1 ky k12 k22 lambda r tau;

B11=((((1-lambda)/lambda)^2)*k12^(-1)+(ky^(-1))/(lambda^2))^(-1)/(((((1-
lambda)/lambda)^2)*k12^(-1)+(ky^(-1))/(lambda^2))^(-1)+k1);

B21=((((1-lambda)/lambda)^2)*k22^(-1)+(ky^(-1))/(lambda^2))^(-1)/(((((1-
lambda)/lambda)^2)*k22^(-1)+(ky^(-1))/(lambda^2))^(-1)+k1);

B12=((((lambda)/(1-lambda))^2)*k1^(-1)+(ky^(-1))/((1-lambda)^2))^(-1)/(((((lambda)/(1-
lambda))^2)*k1^(-1)+(ky^(-1))/((1-lambda)^2))^(-1)+k12);

B22=((((lambda)/(1-lambda))^2)*k1^(-1)+(ky^(-1))/((1-lambda)^2))^(-1)/(((((lambda)/(1-
lambda))^2)*k1^(-1)+(ky^(-1))/((1-lambda)^2))^(-1)+k22);

psi11=(B11*r - r - B11 + B22*r - B11*B22*r + B12*B21*r + B12*lambda*r - B22*lambda*r +
B11*B22*lambda*r - B12*B21*lambda*r + 1)/(B21*r - r + B22*r + B11*lambda*r +
B12*lambda*r - B21*lambda*r - B22*lambda*r + 1);

psi21=(B21*r - r - B21 + B22*r + B12*lambda*r - B22*lambda*r + B11*B22*lambda*r -
B12*B21*lambda*r + 1)/(B21*r - r + B22*r + B11*lambda*r + B12*lambda*r - B21*lambda*r
- B22*lambda*r + 1);

psi12=((lambda - 1)*(B11 - B11*r + B21*r + B11*B22*r - B12*B21*r + B11*lambda*r -
B21*lambda*r - B11*B22*lambda*r + B12*B21*lambda*r))/(lambda*(B21*r - r + B22*r +
B11*lambda*r + B12*lambda*r - B21*lambda*r - B22*lambda*r + 1));

psi22=((lambda - 1)*(B21 + B11*lambda*r - B21*lambda*r - B11*B22*lambda*r +
B12*B21*lambda*r))/(lambda*(B21*r - r + B22*r + B11*lambda*r + B12*lambda*r -
B21*lambda*r - B22*lambda*r + 1));

phi12=(B12*r - r - B12 + B21*r + B11*B22*r - B12*B21*r + B11*lambda*r - B21*lambda*r -
B11*B22*lambda*r + B12*B21*lambda*r + 1)/(B21*r - r + B22*r + B11*lambda*r +
B12*lambda*r - B21*lambda*r - B22*lambda*r + 1);

phi22=(B21*r - r - B22 + B22*r + B11*lambda*r - B21*lambda*r - B11*B22*lambda*r +
B12*B21*lambda*r + 1)/(B21*r - r + B22*r + B11*lambda*r + B12*lambda*r - B21*lambda*r
- B22*lambda*r + 1);
```

```
phi11=(lambda*(B12 - B12*r + B22*r - B11*B22*r + B12*B21*r + B12*lambda*r -
B22*lambda*r + B11*B22*lambda*r - B12*B21*lambda*r))/((lambda - 1)*(B21*r - r + B22*r
+ B11*lambda*r + B12*lambda*r - B21*lambda*r - B22*lambda*r + 1));

phi21=(lambda*(B22 + B12*lambda*r - B22*lambda*r + B11*B22*lambda*r -
B12*B21*lambda*r))/((lambda - 1)*(B21*r - r + B22*r + B11*lambda*r + B12*lambda*r -
B21*lambda*r - B22*lambda*r + 1));

psi1=(B11 - B11*r + B21*r + B11*B22*r - B12*B21*r + B11*lambda*r - B21*lambda*r -
B11*B22*lambda*r + B12*B21*lambda*r)/(lambda*(B21*r - r + B22*r + B11*lambda*r +
B12*lambda*r - B21*lambda*r - B22*lambda*r + 1));

psi2=(B21 + B11*lambda*r - B21*lambda*r - B11*B22*lambda*r +
B12*B21*lambda*r)/(lambda*(B21*r - r + B22*r + B11*lambda*r + B12*lambda*r -
B21*lambda*r - B22*lambda*r + 1));

phi1=-(B12 - B12*r + B22*r - B11*B22*r + B12*B21*r + B12*lambda*r - B22*lambda*r +
B11*B22*lambda*r - B12*B21*lambda*r)/((lambda - 1)*(B21*r - r + B22*r + B11*lambda*r +
B12*lambda*r - B21*lambda*r - B22*lambda*r + 1));

phi2=-(B22 + B12*lambda*r - B22*lambda*r + B11*B22*lambda*r -
B12*B21*lambda*r)/((lambda - 1)*(B21*r - r + B22*r + B11*lambda*r + B12*lambda*r -
B21*lambda*r - B22*lambda*r + 1));

Wealth=-((lambda*(psi11^2+phi11^2)+(1-
lambda)*(psi21^2+phi21^2))*(1/k1)+(lambda*(psi1^2+phi1^2)+(1-
lambda)*(psi2^2+phi2^2))*(1/ky)+lambda*(psi12^2+phi12^2)*(1/k12)+(1-
lambda)*(psi22^2+phi22^2)*(1/k22));

dW_ky=diff(Wealth,ky);  % partial derivative of wealth with respect to the precision of the
public signal %


% Quantitative Results %

k1=1;  % normalization %
k22=k1;  % our case %
r1=[0.6;0.7;0.8;0.9];  % cases considered for different values of r %
ng=51;  % number of grid points %
g=linspace(0.01,1,ng)';  % equally spaced grid points for plotting the results %

lambda=0.5;
H_5=zeros(ng,4);  % defines \tau for each grid point (row) and for each value of r (columns) %
for j=1:4
    for i=1:ng
        k12=k1*g(i);
        r=r1(j);
        ky=(2*r-1)*(1-r)*k1*2*tau;
        f=subs(dW_ky);  % f is a function of \tau now %
        v=solve(f == 0, tau);  % solve for \tau when there is no social value of public information
%
```

```
        t=v(v == real(v));  % there are several roots (one is negative real, complex as well), choose
the positive real %
        H_5(i,j)=max(t);  % plug this root into the matrix of results %
    end;
end;

% Do the same procedure for other values of \lambda %

lambda=0.6;
H_6=zeros(ng,4);
for j=1:4
   for i=1:ng
      k12=k1*g(i);
      r=r1(j);
      ky=(2*r-1)*(1-r)*k1*2*tau;
      f=subs(dW_ky);
      v=solve(f == 0, tau);
      t=v(v == real(v));
      H_6(i,j)=max(t);
   end;
end;

lambda=0.7;
H_7=zeros(ng,4);
for j=1:4
   for i=1:ng
      k12=k1*g(i);
      r=r1(j);
      ky=(2*r-1)*(1-r)*k1*2*tau;
      f=subs(dW_ky);
      v=solve(f == 0, tau);
      t=v(v == real(v));
      H_7(i,j)=max(t);
   end;
end;

lambda=0.8;
H_8=zeros(ng,4);
for j=1:4
   for i=1:ng
      k12=k1*g(i);
      r=r1(j);
      ky=(2*r-1)*(1-r)*k1*2*tau;
      f=subs(dW_ky);
      v=solve(f == 0, tau);
      t=v(v == real(v));
      H_8(i,j)=max(t);
   end;
end;

lambda=0.9;
H_9=zeros(ng,4);
```

```
for j=1:4
    for i=1:ng
        k12=k1*g(i);
        r=r1(j);
        ky=(2*r-1)*(1-r)*k1*2*tau;
        f=subs(dW_ky);
        v=solve(f == 0, tau);
        t=v(v == real(v));
        H_9(i,j)=max(t);
    end;
end;
```

Plotting results

```
% Indifference curve %

ms=repmat(0.5,ng,1);
H1_5=H_5(:,1);
H2_5=H_5(:,2);
H3_5=H_5(:,3);
H4_5=H_5(:,4);
% create figure %
figure1 = figure('Name','Effects of r on welfare: \lambda=0.5');
% create subplot %
subplot1 = subplot(1,1,1,'Parent',figure1);
title('\lambda=0.5')
box(subplot1,'on');
hold(subplot1,'all');
% create multiple lines using matrix input to plot %
plot1 = plot(g,[H1_5, H2_5, H3_5, H4_5, ms],'Parent',subplot1);
set(plot1(1),'Color',[0 0 0],'linewidth',3,'DisplayName','r=0.6');
set(plot1(2),'Color',[1 0 0],'linewidth',3,'DisplayName','r=0.7');
set(plot1(3),'Color',[0 1 0],'linewidth',3,'DisplayName','r=0.8');
set(plot1(4),'Color',[0 0 1],'linewidth',3,'DisplayName','r=0.9');
set(plot1(5),'Color',[0.5 0.5
0.5],'Marker','.','LineStyle','none','linewidth',3,'DisplayName','Morris&Shin (2002) benchmark');
% create xlabel %
xlabel('Inverse dispersion in the precision of private signals: k_{1x}^2/k_{2x}^2');
% create ylabel %
ylabel('\tau');
% create legends %
legend1 = legend(subplot1,'show');
set(legend1,'Location','Best');

% Effects of r on welfare for different \lambda %

H1_6=H_6(:,1);
H2_6=H_6(:,2);
H3_6=H_6(:,3);
H4_6=H_6(:,4);
figure1 = figure('Name','Effects of r on welfare');
```

```
subplot1 = subplot(2,2,1,'Parent',figure1);
title('\lambda=0.6')
box(subplot1,'on');
hold(subplot1,'all');
plot1 = plot(g,[H1_6, H2_6, H3_6, H4_6],'Parent',subplot1);
set(plot1(1),'Color',[0 0 0],'linewidth',2,'DisplayName','r=0.6');
set(plot1(2),'Color',[1 0 0],'linewidth',2,'DisplayName','r=0.7');
set(plot1(3),'Color',[0 1 0],'linewidth',2,'DisplayName','r=0.8');
set(plot1(4),'Color',[0 0 1],'linewidth',2,'DisplayName','r=0.9');
xlabel('Inverse dispersion in the precision of private signals: k_{1x}^2/k_{2x}^2');
ylabel('\tau');
legend1 = legend(subplot1,'show');
set(legend1,'Location','Best');

H1_7=H_7(:,1);
H2_7=H_7(:,2);
H3_7=H_7(:,3);
H4_7=H_7(:,4);
subplot1 = subplot(2,2,2,'Parent',figure1);
title('\lambda=0.7')
box(subplot1,'on');
hold(subplot1,'all');
plot1 = plot(g,[H1_7, H2_7, H3_7, H4_7],'Parent',subplot1);
set(plot1(1),'Color',[0 0 0],'linewidth',2,'DisplayName','r=0.6');
set(plot1(2),'Color',[1 0 0],'linewidth',2,'DisplayName','r=0.7');
set(plot1(3),'Color',[0 1 0],'linewidth',2,'DisplayName','r=0.8');
set(plot1(4),'Color',[0 0 1],'linewidth',2,'DisplayName','r=0.9');
xlabel('Inverse dispersion in the precision of private signals: k_{1x}^2/k_{2x}^2');
ylabel('\tau');
legend1 = legend(subplot1,'show');
set(legend1,'Location','Best');

H1_8=H_8(:,1);
H2_8=H_8(:,2);
H3_8=H_8(:,3);
H4_8=H_8(:,4);
subplot1 = subplot(2,2,3,'Parent',figure1);
title('\lambda=0.8')
box(subplot1,'on');
hold(subplot1,'all');
plot1 = plot(g,[H1_8, H2_8, H3_8, H4_8],'Parent',subplot1);
set(plot1(1),'Color',[0 0 0],'linewidth',2,'DisplayName','r=0.6');
set(plot1(2),'Color',[1 0 0],'linewidth',2,'DisplayName','r=0.7');
set(plot1(3),'Color',[0 1 0],'linewidth',2,'DisplayName','r=0.8');
set(plot1(4),'Color',[0 0 1],'linewidth',2,'DisplayName','r=0.9');
xlabel('Inverse dispersion in the precision of private signals: k_{1x}^2/k_{2x}^2');
ylabel('\tau');
legend1 = legend(subplot1,'show');
set(legend1,'Location','Best');

H1_9=H_9(:,1);
H2_9=H_9(:,2);
```

```
H3_9=H_9(:,3);
H4_9=H_9(:,4);
subplot1 = subplot(2,2,4,'Parent',figure1);
title('\lambda=0.9')
box(subplot1,'on');
hold(subplot1,'all');
plot1 = plot(g,[H1_9, H2_9, H3_9, H4_9],'Parent',subplot1);
set(plot1(1),'Color',[0 0 0],'linewidth',2,'DisplayName','r=0.6');
set(plot1(2),'Color',[1 0 0],'linewidth',2,'DisplayName','r=0.7');
set(plot1(3),'Color',[0 1 0],'linewidth',2,'DisplayName','r=0.8');
set(plot1(4),'Color',[0 0 1],'linewidth',2,'DisplayName','r=0.9');
xlabel('Inverse dispersion in the precision of private signals: k_{1x}^2/k_{2x}^2');
ylabel('\tau');
legend1 = legend(subplot1,'show');
set(legend1,'Location','Best');


% Effects of \lambda on welfare for different r %

L1_6=H_5(:,1);
L2_6=H_6(:,1);
L3_6=H_7(:,1);
L4_6=H_8(:,1);
L5_6=H_9(:,1);
figure1 = figure('Name','Effects of \lambda on welfare');
subplot1 = subplot(2,2,1,'Parent',figure1);
title('r=0.6')
box(subplot1,'on');
hold(subplot1,'all');
plot1 = plot(g,[L1_6, L2_6, L3_6, L4_6, L5_6],'Parent',subplot1);
set(plot1(1),'Color',[0 0 0],'linewidth',2,'DisplayName','\lambda=0.5');
set(plot1(2),'Color',[1 0 0],'linewidth',2,'DisplayName','\lambda=0.6');
set(plot1(3),'Color',[0 1 0],'linewidth',2,'DisplayName','\lambda=0.7');
set(plot1(4),'Color',[0 0 1],'linewidth',2,'DisplayName','\lambda=0.8');
set(plot1(5),'Color',[1 0.5 0],'linewidth',2,'DisplayName','\lambda=0.9');
xlabel('Inverse dispersion in the precision of private signals: k_{1x}^2/k_{2x}^2');
ylabel('\tau');
legend1 = legend(subplot1,'show');
set(legend1,'Location','Best');

L1_7=H_5(:,2);
L2_7=H_6(:,2);
L3_7=H_7(:,2);
L4_7=H_8(:,2);
L5_7=H_9(:,2);
subplot1 = subplot(2,2,2,'Parent',figure1);
title('r=0.7')
box(subplot1,'on');
hold(subplot1,'all');
plot1 = plot(g,[L1_7, L2_7, L3_7, L4_7, L5_7],'Parent',subplot1);
set(plot1(1),'Color',[0 0 0],'linewidth',2,'DisplayName','\lambda=0.5');
set(plot1(2),'Color',[1 0 0],'linewidth',2,'DisplayName','\lambda=0.6');
```

```
set(plot1(3),'Color',[0 1 0],'linewidth',2,'DisplayName','\lambda=0.7');
set(plot1(4),'Color',[0 0 1],'linewidth',2,'DisplayName','\lambda=0.8');
set(plot1(5),'Color',[1 0.5 0],'linewidth',2,'DisplayName','\lambda=0.9');
xlabel('Inverse dispersion in the precision of private signals: k_{1x}^2/k_{2x}^2');
ylabel('\tau');
legend1 = legend(subplot1,'show');
set(legend1,'Location','Best');

L1_8=H_5(:,3);
L2_8=H_6(:,3);
L3_8=H_7(:,3);
L4_8=H_8(:,3);
L5_8=H_9(:,3);
subplot1 = subplot(2,2,3,'Parent',figure1);
title('r=0.8')
box(subplot1,'on');
hold(subplot1,'all');
plot1 = plot(g,[L1_8, L2_8, L3_8, L4_8, L5_8],'Parent',subplot1);
set(plot1(1),'Color',[0 0 0],'linewidth',2,'DisplayName','\lambda=0.5');
set(plot1(2),'Color',[1 0 0],'linewidth',2,'DisplayName','\lambda=0.6');
set(plot1(3),'Color',[0 1 0],'linewidth',2,'DisplayName','\lambda=0.7');
set(plot1(4),'Color',[0 0 1],'linewidth',2,'DisplayName','\lambda=0.8');
set(plot1(5),'Color',[1 0.5 0],'linewidth',2,'DisplayName','\lambda=0.9');
xlabel('Inverse dispersion in the precision of private signals: k_{1x}^2/k_{2x}^2');
ylabel('\tau');
legend1 = legend(subplot1,'show');
set(legend1,'Location','Best');

L1_9=H_5(:,4);
L2_9=H_6(:,4);
L3_9=H_7(:,4);
L4_9=H_8(:,4);
L5_9=H_9(:,4);
subplot1 = subplot(2,2,4,'Parent',figure1);
title('r=0.9')
box(subplot1,'on');
hold(subplot1,'all');
plot1 = plot(g,[L1_9, L2_9, L3_9, L4_9, L5_9],'Parent',subplot1);
set(plot1(1),'Color',[0 0 0],'linewidth',2,'DisplayName','\lambda=0.5');
set(plot1(2),'Color',[1 0 0],'linewidth',2,'DisplayName','\lambda=0.6');
set(plot1(3),'Color',[0 1 0],'linewidth',2,'DisplayName','\lambda=0.7');
set(plot1(4),'Color',[0 0 1],'linewidth',2,'DisplayName','\lambda=0.8');
set(plot1(5),'Color',[1 0.5 0],'linewidth',2,'DisplayName','\lambda=0.9');
xlabel('Inverse dispersion in the precision of private signals: k_{1x}^2/k_{2x}^2');
ylabel('\tau');
legend1 = legend(subplot1,'show');
set(legend1,'Location','Best');
```